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## The Spin Structure of the Proton

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It is shown that the proton “spin crisis” or “spin puzzle” can be understood by the relativistic effect of quark transversal motions due to the Melosh-Wigner rotation. The quark helicity  $\Delta q$  measured in polarized deep inelastic scattering is actually the quark spin in the infinite momentum frame or in the light-cone formalism, and it is different from the quark spin in the nucleon rest frame or in the quark model. The flavor asymmetry of the Melosh-Wigner effect for the valence  $u$  and  $d$  quarks and the intrinsic sea  $q\bar{q}$  pairs are also the important ingredients in a SU(6) quark-spectator-diquark model framework to understand the “spin puzzle”. Such a picture of the spin structure can be tested by use of several simple relations to measure the quark spin distributions in the quark model.

### §1. The proton “spin crisis”

The spin structure of the proton has received attention in the particle physics society for a decade, and there has been a vast number of theoretical and experimental investigations. Parton sum rules and similar relations played important roles in the establishment of the quark-parton picture for nucleons in deep inelastic scattering. Thus any violation of a parton sum rule is of essential importance to reveal possible new content concerning our understanding of the underlying quark-gluon structure of hadrons. From the SU(6) quark model one would expect that the spin of the proton is fully provided by the valence quark spins. Therefore the observation of the Ellis-Jaffe sum rule violation received extensive attention by its implication that the sum of the quark helicities is much smaller than the proton spin. The EMC result of a much smaller integrated spin-dependent structure function data than that expected from the Ellis-Jaffe sum rule triggered the proton “spin crisis”, i.e., the intriguing question of how the spin of the proton is distributed among its quark spin, gluon spin and orbital angular momentum<sup>1)</sup>. It is commonly taken for granted that the EMC result implies that there must be some contribution due to gluon polarization or orbital angular momentum to the proton spin. It will be reported here based on previous works<sup>2)–6)</sup>, however, that the the proton spin problem raised by the Ellis-Jaffe sum rule violation does not in conflict with the SU(6) quark model in which the spin of the proton, when viewed in its rest reference frame, is provided by the vector sum of the quark spins, provided that the relativistic effect from the quark transversal motions<sup>2), 3), 4)</sup>, the flavor asymmetry between the  $u$  and  $d$  valence quarks<sup>3)</sup>, and the intrinsic quark-antiquark pairs generated by the non-perturbative

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meson-baryon fluctuations of the nucleon sea<sup>7)</sup> are taken into account.

## §2. The Melosh-Wigner rotation

As it is known, spin is essentially a relativistic notion associated with the space-time symmetry of Poincaré. The conventional 3-vector spin  $\mathbf{s}$  of a moving particle with finite mass  $m$  and 4-momentum  $p_\mu$  can be defined by transforming its Pauli-Lubánski 4-vector  $\omega_\mu = 1/2 J^{\rho\sigma} P^\nu \epsilon_{\nu\rho\sigma\mu}$  to its rest frame via a non-rotation Lorentz boost  $L(p)$  which satisfies  $L(p)p = (m, \mathbf{0})$ , by  $(0, \mathbf{s}) = L(p)\omega/m$ . Under an arbitrary Lorentz transformation, a particle state with spin  $\mathbf{s}$  and 4-momentum  $p_\mu$  will transform to the state with spin  $\mathbf{s}'$  and 4-momentum  $p'_\mu$ ,

$$\mathbf{s}' = R_\omega(\mathbf{\Lambda}, p)\mathbf{s}, \quad p' = \mathbf{\Lambda}p, \quad (2.1)$$

where  $R_\omega(\mathbf{\Lambda}, p) = L(p')\mathbf{\Lambda}L^{-1}(p)$  is a pure rotation known as Wigner rotation. When a composite system is transformed from one frame to another one, the spin of each constituent will undergo a Wigner rotation. These spin rotations are not necessarily the same since the constituents have different internal motion. In consequence, the sum of the constituent's spin is not Lorentz invariant<sup>2)</sup>.

The key points for understanding the proton spin puzzle lie in the facts that the vector sum of the constituent spins for a composite system is not Lorentz invariant by taking into account the relativistic effect of Wigner rotation, and that it is in the infinite momentum frame the small EMC result was interpreted as an indication that quarks carry a small amount of the total spin of the proton. We call the Wigner rotation from an ordinary frame to the infinite momentum frame the Melosh-Wigner rotation. From the first fact we know that the vector spin structure of hadrons could be quite different in different frames from relativistic viewpoint. We thus can naturally understand the proton "spin crisis" because there is no need to require that the sum of the quark spins is equal to the spin of the proton in the infinite momentum frame, even if the vector sum of the quark spins equals to the proton spin in the rest frame<sup>2)</sup>.

The effect due to the Melosh-Wigner rotation can be best understood from the light-cone spin structure of the pion. It has been shown<sup>8)</sup> that there are higher helicity ( $\lambda_1 + \lambda_2 = \pm 1$ ) components in the light-cone spin space wavefunction for the pion besides the usual helicity ( $\lambda_1 + \lambda_2 = 0$ ) components. Therefore the light-cone wavefunction for the lowest valence state of pion can be expressed as

$$\begin{aligned} |\psi_{q\bar{q}}^\pi\rangle = & \psi(x, \vec{k}_\perp, \uparrow, \downarrow) |\uparrow\downarrow\rangle + \psi(x, \vec{k}_\perp, \downarrow, \uparrow) |\downarrow\uparrow\rangle \\ & + \psi(x, \vec{k}_\perp, \uparrow, \uparrow) |\uparrow\uparrow\rangle + \psi(x, \vec{k}_\perp, \downarrow, \downarrow) |\downarrow\downarrow\rangle, \end{aligned} \quad (2.2)$$

It is interesting to notice that the light-cone wave function (2.2) is the correct pion spin wave function since it is an eigenstate of the total spin operator  $(\hat{S}^F)^2$  in the light-cone formalism<sup>8)</sup>.

It is thus necessary to clarify what is meant by the quantity  $\Delta q$  defined by  $\Delta q \cdot S_\mu = \langle P, S | \bar{q} \gamma_\mu \gamma_5 q | P, S \rangle$ , where  $S_\mu$  is the proton polarization vector.  $\Delta q$  can be calculated from  $\Delta q = \langle P, S | \bar{q} \gamma^+ \gamma_5 q | P, S \rangle$  since the instantaneous fermion lines do

not contribute to the + component. One can easily prove, by expressing the quark wave functions in terms of light-cone Dirac spinors (i.e., the quark spin states in the infinite momentum frame), that

$$\Delta q = \int_0^1 dx [q^\uparrow(x) - q^\downarrow(x)], \quad (2.3)$$

where  $q^\uparrow(x)$  and  $q^\downarrow(x)$  are the probabilities of finding, in the proton infinite momentum frame, a quark or antiquark of flavor  $q$  with fraction  $x$  of the proton longitudinal momentum and with polarization parallel or antiparallel to the proton spin, respectively. However, if one expresses the quark wave functions in terms of conventional instant form Dirac spinors (i.e., the quark spin state in the proton rest frame), it can be found, that

$$\Delta q = \int d^3\vec{p} M_q [q^\uparrow(p) - q^\downarrow(p)] = \langle M_q \rangle \Delta q_{QM}, \quad (2.4)$$

with

$$M_q = [(p_0 + p_3 + m)^2 - \vec{p}_\perp^2] / [2(p_0 + p_3)(m + p_0)] \quad (2.5)$$

being the contribution from the relativistic effect due to the quark transversal motions,  $q^\uparrow(p)$  and  $q^\downarrow(p)$  being the probabilities of finding, in the proton rest frame, a quark or antiquark of flavor  $q$  with rest mass  $m$  and momentum  $p_\mu$  and with spin parallel or antiparallel to the proton spin respectively, and  $\Delta q_{QM} = \int d^3\vec{p} [q^\uparrow(p) - q^\downarrow(p)]$  being the net spin vector sum of quark flavor  $q$  parallel to the proton spin in the rest frame. Thus one sees that the quantity  $\Delta q$  should be interpreted as the net spin polarization in the infinite momentum frame if one properly considers the relativistic effect due to internal quark transversal motions<sup>2)</sup>.

Since  $\langle M_q \rangle$ , the average contribution from the relativistic effect due to internal transversal motions of quark flavor  $q$ , ranges from 0 to 1 (or more properly, it should be around 0.75 for light flavor quarks and approaches 1 for heavy flavor quarks), and  $\Delta q_{QM}$ , the net spin vector polarization of quark flavor  $q$  parallel to the proton spin in the proton rest frame, is related to the quantity  $\Delta q$  by the relation  $\Delta q_{QM} = \Delta q / \langle M_q \rangle$ , we have sufficient freedom to make the naive quark model spin sum rule, i.e.,  $\Delta u_{QM} + \Delta d_{QM} + \Delta s_{QM} = 1$ , satisfied while still preserving the values of  $\Delta u$ ,  $\Delta d$  and  $\Delta s$  as parametrized from experimental data in appropriate explanations. Thereby we can understand the “spin crisis” simply because the quantity  $\Delta \Sigma = \Delta u + \Delta d + \Delta s$  does not represent, in a strict sense, the vector sum of the spin carried by the quarks in the naive quark model. It is possible that the value of  $\Delta \Sigma = \Delta u + \Delta d + \Delta s$  is small whereas the spin sum rule

$$\Delta u_{QM} + \Delta d_{QM} + \Delta s_{QM} = 1 \quad (2.6)$$

for the naive quark model still holds, though the realistic situation may be complicated.

### §3. A light-cone quark-spectator-diquark model for nucleons

From the impulse approximation picture of deep inelastic scattering, one can calculate the valence quark distributions in the quark-diquark model where the single

valence quark is the scattered parton and the non-interacting diquark serves to provide the quantum number of the spectator<sup>3)</sup>. From the nucleon wave function of the SU(6) quark-spectator-diquark model<sup>3)</sup>, we get the unpolarized quark distributions

$$\begin{aligned} u_v(x) &= a_S(x)/2 + a_V(x)/6; \\ d_v(x) &= a_V(x)/3, \end{aligned} \quad (3.1)$$

where  $a_D(x)$  ( $D = S$  or  $V$  representing the vector ( $V$ ) or scalar ( $S$ ) diquarks) is normalized such that  $\int_0^1 dx a_D(x) = 3$  and denotes the amplitude for the quark  $q$  is scattered while the spectator is in the diquark state  $D$ . Therefore we can write, by assuming the isospin symmetry between the proton and the neutron, the unpolarized structure functions for nucleons,

$$\begin{aligned} F_2^p(x) &= xs(x) + \frac{2}{9}xa_S(x) + \frac{1}{9}xa_V(x); \\ F_2^n(x) &= xs(x) + \frac{1}{18}xa_S(x) + \frac{1}{6}xa_V(x), \end{aligned} \quad (3.2)$$

where  $s(x)$  denotes the contribution from the sea.

Exact SU(6) symmetry provides the relation  $a_S(x) = a_V(x)$  which implies the valence flavor symmetry  $u_v(x) = 2d_v(x)$ . This gives the prediction  $F_2^n(x)/F_2^p(x) \geq 2/3$  for all  $x$  and is ruled out by the experimental observation  $F_2^n(x)/F_2^p(x) < 1/2$  for  $x \rightarrow 1$ . It has been a well established fact that the valence flavor symmetry  $u_v(x) = 2d_v(x)$  does not hold and the explicit  $u_v(x)$  and  $d_v(x)$  can be parameterized from the combined experimental data from deep inelastic scatterings of electron (muon) and neutrino (anti-neutrino) on the proton and the neutron *et al.*. In this sense, any theoretical calculation of quark distributions should reflect the flavor asymmetry between the valence  $u$  and  $d$  quarks in a reasonable picture. It has been shown<sup>3)</sup> that the mass difference between the scalar and vector spectators can reproduce the up and down valence quark asymmetry that accounts for the observed ratio  $F_2^n(x)/F_2^p(x)$  at large  $x$ .

The amplitude for the quark  $q$  is scattered while the spectator in the spin state  $D$  can be written as

$$a_D(x) \propto \int [d^2\mathbf{k}_\perp] |\varphi_D(x, \mathbf{k}_\perp)|^2. \quad (3.3)$$

We adopt the Brodsky-Huang-Lepage prescription for the light-cone momentum space wave function<sup>9)</sup> of the quark-spectator

$$\varphi_D(x, \mathbf{k}_\perp) = A_D \exp\left\{-\frac{1}{8\beta_D^2} \left[ \frac{m_q^2 + \mathbf{k}_\perp^2}{x} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1-x} \right]\right\}, \quad (3.4)$$

where  $\mathbf{k}_\perp$  is the internal quark transversal momentum,  $m_q$  and  $m_D$  are the masses for the quark  $q$  and spectator  $D$ , and  $\beta_D$  is the harmonic oscillator scale parameter. The values of the parameters  $\beta_D$ ,  $m_q$  and  $m_D$  can be adjusted by fitting the hadron properties such as the electromagnetic form factors, the mean charge radiuses, and the weak decay constants *et al.* in the relativistic light-cone quark model. We simply adopt  $m_q = 330$  MeV and  $\beta_D = 330$  MeV. The masses of the scalar and vector spectators should be different taking into account the spin force from color magnetism, and we choose, e.g.,  $m_S = 600$  MeV and  $m_V = 900$  MeV as estimated

to explain the  $N-\Delta$  mass difference. The mass difference between the scalar and vector spectators causes difference between  $a_S(x)$  and  $a_V(x)$  and thus the flavor asymmetry between the valence quark distribution functions  $u_v(x)$  and  $d_v(x)$ . The calculated results<sup>3)</sup> are in reasonable agreement with the experimental data and this supports the quark-spectator picture of deep inelastic scattering in which the difference between the scalar and vector spectators is important to reproduce the explicit SU(6) symmetry breaking while the bulk SU(6) symmetry of the quark model still holds.

For the polarized quark distributions, we take into account the contribution from the Wigner rotation<sup>2)</sup>. In the light-cone or quark-parton descriptions,  $\Delta q(x) = q^\uparrow(x) - q^\downarrow(x)$ , where  $q^\uparrow(x)$  and  $q^\downarrow(x)$  are the probability of finding a quark or antiquark with longitudinal momentum fraction  $x$  and polarization parallel or antiparallel to the proton helicity in the infinite momentum frame. However, in the proton rest frame, one finds,

$$\Delta q(x) = \int [d^2\mathbf{k}_\perp] W_D(x, \mathbf{k}_\perp) [q_{s_z=\frac{1}{2}}(x, \mathbf{k}_\perp) - q_{s_z=-\frac{1}{2}}(x, \mathbf{k}_\perp)], \quad (3.5)$$

with

$$W_D(x, \mathbf{k}_\perp) = [(k^+ + m)^2 - \mathbf{k}_\perp^2] / [(k^+ + m)^2 + \mathbf{k}_\perp^2] \quad (3.6)$$

being the contribution from the relativistic effect due to the quark transversal motions,  $q_{s_z=\frac{1}{2}}(x, \mathbf{k}_\perp)$  and  $q_{s_z=-\frac{1}{2}}(x, \mathbf{k}_\perp)$  being the probability of finding a quark and antiquark with rest mass  $m$  and with spin parallel and anti-parallel to the rest proton spin, and  $k^+ = x\mathcal{M}$  where  $\mathcal{M} = \frac{m_q^2 + \mathbf{k}_\perp^2}{x} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1-x}$ . The Wigner rotation factor  $W_D(x, \mathbf{k}_\perp)$  ranges from 0 to 1; thus  $\Delta q$  measured in polarized deep inelastic scattering cannot be identified with the spin carried by each quark flavor in the proton rest frame.

The spin distribution probabilities in the quark-diquark model read<sup>3)</sup>

$$\begin{aligned} u_V^\uparrow &= \frac{1}{18}; & u_V^\downarrow &= \frac{2}{18}; & d_V^\uparrow &= \frac{2}{18}; & d_V^\downarrow &= \frac{4}{18}; \\ u_S^\uparrow &= \frac{1}{2}; & u_S^\downarrow &= 0; & d_S^\uparrow &= 0; & d_S^\downarrow &= 0. \end{aligned} \quad (3.7)$$

Taking into account the Melosh-Wigner rotation, we can write the quark helicity distributions for the  $u$  and  $d$  quarks

$$\begin{aligned} \Delta u_v(x) &= u_v^\uparrow(x) - u_v^\downarrow(x) = -\frac{1}{18}a_V(x)W_V(x) + \frac{1}{2}a_S(x)W_S(x); \\ \Delta d_v(x) &= d_v^\uparrow(x) - d_v^\downarrow(x) = -\frac{1}{9}a_V(x)W_V(x), \end{aligned} \quad (3.8)$$

where  $W_D(x)$  is the correction factor due the Melosh-Wigner rotation. From Eq. (3.1) one gets

$$\begin{aligned} a_S(x) &= 2u_v(x) - d_v(x); \\ a_V(x) &= 3d_v(x). \end{aligned} \quad (3.9)$$

Combining Eqs. (3.8) and (3.9) we have

$$\begin{aligned} \Delta u_v(x) &= [u_v(x) - \frac{1}{2}d_v(x)]W_S(x) - \frac{1}{6}d_v(x)W_V(x); \\ \Delta d_v(x) &= -\frac{1}{3}d_v(x)W_V(x). \end{aligned} \quad (3.10)$$

Thus we arrive at simple relations between the polarized and unpolarized quark distributions for the valence  $u$  and  $d$  quarks. We can calculate the quark helicity distributions  $\Delta u_v(x)$  and  $\Delta d_v(x)$  from the unpolarized quark distributions  $u_v(x)$  and  $d_v(x)$  by relations (3.10), once the detailed  $x$ -dependent Wigner rotation factor  $W_D(x)$  is known. On the other hand, we can also use relations (3.10) to study  $W_S(x)$  and  $W_V(x)$ , once there are good quark distributions  $u_v(x)$ ,  $d_v(x)$ ,  $\Delta u_v(x)$ , and  $\Delta d_v(x)$  from experiments. From another point of view, the relations (3.10) can be considered as the results of the conventional SU(6) quark model by explicitly taking into account the Wigner rotation effect and the flavor asymmetry introduced by the mass difference between the scalar and vector spectators, thus any evidence for the invalidity of Eq. (3.10) will be useful to reveal new physics beyond the SU(6) quark model.

We calculated the  $x$ -dependent Wigner rotation factor  $W_D(x)$  in the light-cone SU(6) quark-spectator model<sup>3)</sup> and noticed slight asymmetry between  $W_S(x)$  and  $W_V(x)$ . Considering only the valence quark contributions, we can write the spin-dependent structure functions  $g_1^p(x)$  and  $g_1^n(x)$  for the proton and the neutron by

$$\begin{aligned} g_1^p(x) &= \frac{1}{2} \left[ \frac{4}{9} \Delta u_v(x) + \frac{1}{9} \Delta d_v(x) \right] = \frac{1}{18} [(4u_v(x) - 2d_v(x))W_S(x) - d_v(x)W_V(x)]; \\ g_1^n(x) &= \frac{1}{2} \left[ \frac{1}{9} \Delta u_v(x) + \frac{4}{9} \Delta d_v(x) \right] = \frac{1}{36} [(2u_v(x) - d_v(x))W_S(x) - 3d_v(x)W_V(x)]. \end{aligned} \quad (3.11)$$

We found<sup>3)</sup> that the calculated  $A_1^N$  with Wigner rotation are in agreement with the experimental data, at least for  $x \geq 0.1$ . The large asymmetry between  $W_S(x)$  and  $W_V(x)$  has consequence for a better fit of the data.

As we consider only the valence quark contributions to  $g_1^p(x)$  and  $g_1^n(x)$ , we should not expect to fit the Ellis-Jaffe sum data from experiments. This leaves room for additional contributions from sea quarks or other sources. We point out, however, it is possible to reproduce the observed Ellis-Jaffe sums  $\Gamma_1^p = \int_0^1 g_1^p(x) dx$  and  $\Gamma_1^n = \int_0^1 g_1^n(x) dx$  within the light-cone SU(6) quark-spectator model by introducing a large asymmetry between the Wigner rotation factors  $W_S$  and  $W_V$  for the scalar and vector spectators. For example, we need  $\langle W_S \rangle = 0.56$  and  $\langle W_V \rangle = 0.92$  to produce  $\Gamma_1^p = 0.136$  and  $\Gamma_1^n = -0.03$  as observed in experiments. This can be achieved by adopting a large difference between  $\beta_S$  and  $\beta_V$  which should be adjusted by fitting other nucleon properties in the model<sup>10)</sup>. The calculated  $A_1^p(x)$ ,  $A_1^n(x)$ , and  $A_1^d(x)$  are in good agreement with the data<sup>3)</sup>. This may suggest that the explicit SU(6) asymmetry could be also used to explain the EJSR violation (or partially) within a bulk SU(6) symmetry scheme of the quark model, or we take this as a hint for other SU(6) breaking source in addition to the SU(6) quark model.

We showed in the above that the  $u$  and  $d$  asymmetry in the lowest valence component of the nucleon and the Melosh-Wigner rotation effect due to the internal quark transversal motions are important for re-producing the observed ratio  $F_2^n/F_2^p$  and the polarization asymmetries  $A_1^N$  for the proton, neutron, and deuteron. For a better understanding of the origin of polarized sea quarks implied by the violation of the Ellis-Jaffe sum rule, we still need to consider the higher Fock states implied by the non-perturbative meson-baryon fluctuations<sup>7)</sup>. In the light-cone meson-baryon fluctuation model, the net  $d$  quark helicity of the intrinsic  $q\bar{q}$  fluctuation is nega-

tive, whereas the net  $\bar{d}$  antiquark helicity is zero. Therefore the quark/antiquark asymmetry of the  $d\bar{d}$  pairs should be apparent in the  $d$  quark and antiquark helicity distributions. There are now explicit measurements of the helicity distributions for the individual  $u$  and  $d$  valence and sea quarks by SMC<sup>11)</sup> and HERMES<sup>12)</sup>. The helicity distributions for the  $u$  and  $d$  antiquarks are consistent with zero in agreement with the results of the light-cone meson-baryon fluctuation model of intrinsic  $q\bar{q}$  pairs. The calculated quark helicity distributions  $\Delta u_v(x)$  and  $\Delta d_v(x)$  have been compared<sup>3)</sup> with the recent SMC data. The data are still not precise enough for making detailed comparison, but the agreement with  $\Delta u_v(x)$  seems to be good. It seems that the agreement with  $\Delta d_v(x)$  is poor and there is somewhat evidence for additional source of negative helicity contribution to the valence  $d$  quark beyond the conventional quark model. This again supports the light-cone meson-baryon fluctuation model in which the helicity distribution of the intrinsic  $d$  sea quarks  $\Delta d_s(x)$  is negative.

The standard SU(6) quark model gives the constraints  $|\Delta u_v| \leq \frac{4}{3}$  and  $|\Delta d_v| \leq \frac{1}{3}$ . A global fit<sup>13)</sup> of polarized deep inelastic scattering data leads to a value:  $\Delta d = -0.43 \pm 0.03$ . In the light-cone meson-baryon fluctuation model, the antiquark helicity contributions are zero. We thus can consider the empirical values as the helicity contributions  $\Delta q = \Delta q_v + \Delta q_s$  from both the valence  $q_v$  and sea  $q_s$  quarks. Thus the empirical result  $|\Delta d| > \frac{1}{3}$  strongly implies an additional negative contribution  $\Delta d_s$  in the nucleon sea.

#### §4. How to test the picture?

The key point that the light-cone SU(6) quark-diquark model<sup>3)</sup> can give a good description of the experimental observation related to the proton spin quantities relies on the fact that the quark helicity measured in polarized deep inelastic scattering is different from the quark spin in the rest frame of the nucleon or in the quark model<sup>2), 4)</sup>. Thus the observed small value of the quark helicity sum for all quarks is not necessarily in contradiction with the quark model in which the proton spin is provided by the valence quarks. From this sense, there is no serious “spin puzzle” or “spin crisis” as it was first understood. Of course, the sea quark content of the nucleon is complicated and it seems that the baryon-meson fluctuation configuration<sup>7)</sup> composes one important part of the non-perturbative aspects of the nucleon. We should not expect that the valence quarks provide 100% of the proton spin, and the sea quarks and gluons should also contribute some part of the proton spin, thus it is meaningful to design new experimental methods to measure these contributions independently. Useful relations that can be used to measure the quark spin as meant in the quark model and the quark orbital angular momentum from a relativistic viewpoint have been discussed<sup>4), 5)</sup>. It has been pointed out by Schmidt, Soffer, and I that the quark spin distributions  $\Delta q_{QM}(x)$  are connected with the quark helicity distributions  $\Delta q(x)$  and the quark transversity distributions  $\delta q(x)$  by an approximate relation<sup>4)</sup>:

$$\Delta q_{QM}(x) + \Delta q(x) = 2\delta q(x). \quad (4.1)$$

The quark orbital angular momentum  $L_q(x)$  and the quark helicity distribution  $\Delta q(x)$  are also found by Schmidt and I to be connected to the quark model spin distribution  $\Delta q_{QM}(x)$  by a relation<sup>5)</sup>:

$$\Delta q(x)/2 + L_q(x) = \Delta q_{QM}(x)/2, \quad (4.2)$$

which means that one can decompose the quark model spin contribution  $\Delta q_{QM}(x)$  by a quark helicity term  $\Delta q(x)$  *plus* an orbital angular momentum term  $L_q(x)$ . There is also a new relation connecting the quark orbital angular momentum with the measurable quark helicity distribution and transversity distribution<sup>5)</sup>:

$$\Delta q(x) + L_q(x) = \delta q(x), \quad (4.3)$$

from which we may have new sum rules connecting the quark orbital angular momentum with the nucleon axial and tensor charges. The quark transversity and orbital angular momentum distributions have been also calculated in the light-cone SU(6) quark-diquark model<sup>4), 5)</sup>. Thus future measurements of new physical quantities related to the proton spin structure can be used to test whether the framework is correct or not, and detailed predictions and discussions can be found in Refs.<sup>4), 5), 6)</sup>. We point out that one of the predictions of the framework is the small helicity contribution from the anti-quarks and the available experimental data<sup>11), 12)</sup> are consistent with this prediction. This is different from most other works in which a large negative spin contribution from anti-quarks is required to reproduce the observed small quark helicity sum. In our framework the Melosh-Wigner rotation<sup>2), 4)</sup> and the flavor asymmetry of the Melosh-Wigner rotation factors between the  $u$  and  $d$  quarks<sup>3)</sup> are the main reason for the reduction of the quark helicity sum compared to the naive quark model prediction.

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